Grades 9-12 Mathematics Writing Committee

All high school mathematics teachers participated in the high school mathematics curriculum revision process, thereby affording opportunities for teachers to add their knowledge and professional experiences to the process. Teachers on the mathematics writing committee based the curriculum on teacher input and recommendations collected from collaboration efforts. The curriculum writing committee members will act as a resource to their department colleagues. Opportunities to discuss the curriculum implementation, teaching strategies, resources, as well as reflections and concerns will be provided during the 2011-12 school year.

I am grateful to the mathematics curriculum committee members for the many hours and expertise they devoted to writing this curriculum guide. The committee members are commended for their efforts and interest in providing the Park Ridge School District with a thorough high school mathematics curriculum that aligns with the Common Core State Standards for Mathematics.

According to the 2010 Common Core State Standards Initiative, "The Common Core State Mathematical Standards are designed to provide a clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers."

Dr. Cathy Timpone
Director of Curriculum & Instruction

High School Mathematics Curriculum Writing Committee:

Lynne Lupfer revised:
Mathematics Prep, Geometry Honors, Algebra II Honors,
Pre-Calculus, Pre-Calculus Honors, SAT Prep Math curriculum guides

Rachel Berger revised:
Pre-Algebra, Algebra II, Algebra II Honors curriculum guides

Erin Havel revised:
Algebra I, Algebra I Honors curriculum guides

Roseanne Wates revised:
Geometry, Statistics curriculum guides

Dana Caine revised:
Statistics curriculum guide

Anna Marie Schoenkopf revised:
Life Skills Math curriculum guide

Debbie Strammiello revised:
Life Skills Math curriculum guides

Debra Aach, Mathematics Supervisor

Administration:

Troy Lederman, Principal, Park Ridge High School
Dr. Cathy Timpone, Director of Curriculum and Instruction
Dr. Robert Gamper, Superintendent

Board Approved August 27, 2012

PARK RIDGE SCHOOL DISTRICT
Park Ridge, NJ

Mathematics DEPARTMENT, 2011-2012

COURSE: CALCULUS (H) COURSE # 2560
PREREQUISITE: Pre-calculus or Pre-calculus Honors
TIME ALLOCATION: Full Year
TEXTS: Calculus of a Single Variable 7th Ed. Larson, Hostetler, Edwards
COURSE PHILOSOPHY (grades 7-12): This course provides an introduction to calculus of functions of a single variable, including differentiation and integration with applications.

OVERARCHNG ENDURING UNDERSTANDINGS (grade 7-12):
Calculus is the mathematics of change.
The concept of a limit is one of the foundations of calculus.
The derivative is the instantaneous rate of change at a given point.
The integral is a function that can be used to determine the summation of an infinite set.
Differentiation and definite integration are inverse operations.

OVERARCHING ESSENTIAL QUESTIONS (grades 7-12):
How do we calculate instantaneous rates of change?
How does the integral represent the summation of an infinite set?
How do we find the area under a curve, the length of a curve and the volumes of a solid of revolution?

TABLE OF CONTENTS (UNITS OF STUDY):

UNIT ONE: LIMITS AND CONTINUITY

UNIT SUMMARY:

As an overview:

1. The student will learn to find the limit of a function.
2. Limits are the basis of all calculus.
3. The students will learn to determine the limit of a function at any given value and also the continuity of a function.
4. In this unit we will reduce rational functions and find the conjugate of radical expressions. We will also continue previous discussions of domain and range

21ST CENTURY THEMES:

- ☒ Global Awareness
- ☒ Financial, Economic, Business and Entrepreneurial Literacy
- ☐ Civic Literacy
- ☐ Health Literacy

PRIMARY INTERDISCIPLINARY CONNECTIONS:

1. This unit begins our study of the mathematics underlying physics and engineering.

STANDARDS

N-CN. 3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
A-APR. 6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
A-APR7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
A-REI. 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A-REI. 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A-REI.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
F-IF. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example,*

given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences.
9.1.12.B.1 Present resources and data in a format that effectively communicates the meaning of the data and its implications for solving problems, using multiple perspectives.
9.1.12.C.5 Assume a leadership position by guiding the thinking of peers in a direction that leads to successful completion of a challenging task or project.
9.1.12.F.2 Demonstrate a positive work ethic in various settings, including the classroom and during structured learning experiences.
RST.11.3-7
3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.
5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.
6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.
7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.

UNIT ESSENTIAL QUESTIONS:	**UNIT ENDURING UNDERSTANDINGS:**
• What is a limit? • What are the different ways we can calculate a limit? • How do we interpret graphs to calculate the limit? • How do we determine the continuity of a function? • What is the relationship between continuity and limits?	• Functions can approach a value without ever having to reach that exact value. ▪ Continuity implies that the limit exists..

TIME ALLOTMENT	**CONTENT**	**SKILLS**	**ASSESSMENT**
15 Days	• Limit • Continuous • Removable discontinuity • Non-removable discontinuity • Intermediate Value Theorem • Jump discontinuity • One-sided limit	A:Finding Limits 1. Use the properties of limits 2. Find one-sided limits 3. Find Trigonometric limits B: Discussing Continuity 1. Determine continuous functions 2. Identify discontinuous functions a. Find removable discontinuity b. Find essential/non-removable/jump discontinuity c. Determine infinite discontinuity C: Discuss Limits and Continuity 1. Determine relationships between functions	• Tests • Quizzes • *Take It to the Limit* Project

		2. Understand and apply the Intermediate Value Theorem	

TEACHER RESOURCES	**TEACHER NOTES/REFLECTIONS**
▪ Textbook ▪ www.calculus-help.com ▪ The Eagles' *Take It to the Limit* recording ▪ Graphing calculator ▪ SMARTBoard	

UNIT TWO: DERIVATIVES

UNIT SUMMARY:

As an overview:

1. The students will be able to calculate the derivative of a function. This will enable them to determine instantaneous rate of change and the slope of the tangent line to a function at any given point..
2. The derivative of a function is defined as the limit of the slope of the secant line at the difference of the change in x values approaches zero.
3. The derivative is the basis of the mathematics of change.

21ST CENTURY THEMES:	**PRIMARY INTERDISCIPLINARY CONNECTIONS:**
☒ Global Awareness ☒ Financial, Economic, Business and Entrepreneurial Literacy ☐ Civic Literacy ☐ Health Literacy	1. This unit has a direct correlation with physics, economics and engineering. .

STANDARDS

N-Q 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-SSE 1. Interpret expressions that represent a quantity in terms of its context.★
- a. Interpret parts of an expression, such as terms, factors, and coefficients.
- b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

2. Use the structure of an expression to identify ways to rewrite it.
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
- a. Factor a quadratic expression to reveal the zeros of the function it defines.
- b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- c. Use the properties of exponents to transform expressions for exponential functions.

A –APR 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A –CED 1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A -RE I 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
- b. Solve quadratic equations by inspection (e.g., for $x_2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F-IF 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an Element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-TF 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–*x*, □+*x*, and 2□–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

G-CO 1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences.

9.1.12.B.1 Present resources and data in a format that effectively communicates the meaning of the data and its implications for solving problems, using multiple perspectives.

9.1.12.C.5 Assume a leadership position by guiding the thinking of peers in a direction that leads to successful completion of a challenging task or project.

9.1.12.F.2 Demonstrate a positive work ethic in various settings, including the classroom and during structured learning experiences.

RST.11.3-7

3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.

5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.

6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.

7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.

UNIT ESSENTIAL QUESTIONS:	**UNIT ENDURING UNDERSTANDINGS:**
▪ How does the rate of change relate to a derivative? ▪ How does differentiability relate to continuity? ▪ How do we calculate the derivative of all functions? ▪ Can we find the derivative of all functions? ▪ When/How does one know to use the product rule, quotient rule, chain rule? ▪ How do we find higher order derivatives? ▪ When do we use implicit differentiation?	▪ Derivatives are rates of change

TIME ALLOTMENT	CONTENT	SKILLS	ASSESSMENT
45 day	▪ Tangent to a curve ▪ Position function ▪ Velocity function ▪ Acceleration function ▪ Power rule ▪ Product rule ▪ Quotient rule ▪ Chain rule ▪ Implicit differentiation ▪ Higher order derivatives	A: Use the limit definition to find the derivative of a function B: Determine the Differentiability of a function 1. Cusps 2. Corners 3. Infinite oscillations C: Calculate Rates of change 1. Use the Position function to calculate Velocity 2. Compare and contrast Average vs. instantaneous rates of change 3. Calculate Speed and Acceleration D: Apply Rules of Differentiation in appropriate situations 1. Power rule 2. Product rule 3. Quotient rule 4. Derive Trigonometric functions 5. Chain rule a. Use all previous rules incorporated within 6. Apply Implicit differentiation 7. Derive Exponential and logarithmic functions 8. Calculate Higher order derivatives	▪ Tests ▪ Quizzes ▪ Exit Slips ▪ Student Presentations of homework problems at the SmartBoard ▪ *Who Invented Calculus?* webquest

TEACHER RESOURCES	TEACHER NOTES/REFLECTIONS
Textbook www.calculus-help.com *Calculus in Motion* software Graphing Calculator SMARTboard	

UNIT THREE: APPLICATIONS OF THE DERIVATIVE

UNIT SUMMARY:

As an overview:

1. The students will use derivatives to solve real world problems.
2. Having learned how to find the derivative, students will apply their knowledge in real world situations.
3. Students will use multiple problem solving strategies to solve equations, including real life mathematical problems.

21ST CENTURY THEMES:	**PRIMARY INTERDISCIPLINARY CONNECTIONS:**
☒ Global Awareness ☒ Financial, Economic, Business and Entrepreneurial Literacy ☐ Civic Literacy ☐ Health Literacy	This unit has a direct correlation with physics, economics and engineering.

STANDARDS

N-Q 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-SSE 1. Interpret expressions that represent a quantity in terms of its context.⋆

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

2. Use the structure of an expression to identify ways to rewrite it.
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions.

A –APR 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A –CED 1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A -RE I 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x_2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F-IF 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an Element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

4. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-TF 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

5. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–x, □+x, and 2□–x in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

G-CO 1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-GMD 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

G-MG 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost;

9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences.

9.1.12.B.1 Present resources and data in a format that effectively communicates the meaning of the data and its implications for solving problems, using multiple perspectives.

9.1.12.C.5 Assume a leadership position by guiding the thinking of peers in a direction that leads to successful completion of a challenging task or project.

9.1.12.F.2 Demonstrate a positive work ethic in various settings, including the classroom and during structured learning experiences.

RST.11.3-7

3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.

5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.
6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.
7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.

UNIT ESSENTIAL QUESTIONS:	**UNIT ENDURING UNDERSTANDINGS:**
▪ How do we calculate change with respect to time? ▪ When and how do we use the applications of the first derivative, such as Rolle's Theorem, Mean Value Theorem, Extreme Value Theorem and First Derivative Test? ▪ When and how do we use the applications of the second derivative, such as the 2nd Derivative Test? ▪ How do we analyze graphs of the original function, and its first and second derivatives? ▪ How can we sketch the graph of a function using limits and derivatives? ▪ How do we find minimums and maximums of real-world problems?	Derivatives can be used in real-world situations to find rates of change

TIME ALLOTMENT	**CONTENT**	**SKILLS**	**ASSESSMENT**
30 days	▪ Related rates ▪ Extrema ▪ Extreme value theorem ▪ Rolle's theorem ▪ Increasing function ▪ Decreasing function ▪ Concavity ▪ Asymptote ▪ Optimization ▪ Newton's method	A: Calculate and interpret Related Rates B: Calculate Extrema Values 1. Local extrema 2. Absolute extrema 3. Apply the Extreme value theorem C: Use applications of the first derivative 1. Rolle's Theorem 2. Mean Value Theorem 3. Extrema values and critical numbers 4. Increasing and decreasing functions D: Applying the second derivative 1. Concavity 2. Points of inflection 3. Second derivative test E: Apply the first and second derivatives to sketch curves 1. Find Infinite limits a. Find asymptotes i. Vertical	▪ Tests ▪ Quizzes ▪ Exit Slips ▪ Student Presentations ▪ *Tootsie Pop* Lab

		ii. Horizontal iii. Slant 2. Use the original function to find a. Domain/range b. Zeros c. Asymptotes d. Symmetry 3. The first derivative 4. The second derivative F: Analyze Graphs 1. Interpret from original graph to derivative 2. Interpret from derivative graph to the original G: Solve Optimization Problems H: Use Newton's Method	

TEACHER RESOURCES	**TEACHER NOTES/REFLECTIONS**
Textbook *Calculus in Motion* software Graphing Calculator SMARTboard Tootsie Pops Stop Watch	

UNIT FOUR: THE INTEGRAL

UNIT SUMMARY:

As an overview:

1. The students will be able to use the integral to find the area under a curve.
2. Students will use their knowledge of derivatives to find antiderivatives.
3. Students will be able to find indefinite and definite integrals and solve differential equations.

21ST CENTURY THEMES:

- ☒ Global Awareness
- ☒ Financial, Economic, Business and Entrepreneurial Literacy
- ☐ Civic Literacy
- ☐ Health Literacy

PRIMARY INTERDISCIPLINARY CONNECTIONS:

This unit has a direct correlation with physics, economics and engineering.

STANDARDS

N-Q 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-SSE 1. Interpret expressions that represent a quantity in terms of its context.⋆

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

2. Use the structure of an expression to identify ways to rewrite it.
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.⋆

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions.

A –APR 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A –CED 1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A -RE I 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.

Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x_2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F-IF 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an Element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

6. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-TF 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

7. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–x, □+x, and 2□–x in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences.

9.1.12.B.1 Present resources and data in a format that effectively communicates the meaning of the data and its implications for solving problems, using multiple perspectives.

9.1.12.C.5 Assume a leadership position by guiding the thinking of peers in a direction that leads to successful completion of a challenging task or project.

9.1.12.F.2 Demonstrate a positive work ethic in various settings, including the classroom and during structured learning experiences.

RST.11.3-7

3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.

5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.

6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.

7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.

UNIT ESSENTIAL QUESTIONS:	**UNIT ENDURING UNDERSTANDINGS:**
▪ How do we find anti-derivatives of all types of functions? ▪ How do we find the area under the curve using Riemann Sums and the	▪ The integral is the area underneath a curve

Trapezoidal Rule? ▪ How do we find the average value of a function? ▪ How are the 1st and 2nd Fundamental Theorems of Calculus applicable? ▪ How and why do we solve differential equations?	

TIME ALLOTMENT	CONTENT	SKILLS	ASSESSMENT
45 Days	▪ Integration ▪ Riemann sums ▪ First & Second Fundamental theorems of calculus ▪ Trapezoidal rule ▪ Simpson's rule ▪ u-substitution ▪ slope field	A: Determine Anti-derivatives of a function 1. Basic integration rules a. Trigonometric int. B: Calculate the area underneath the curve 1. Basic graphs 2. Riemann sums 3. Trapezoidal Rule** 4. Simpson's Rule** C: Use the First Fundamental Theorem of Calculus 1. Apply the average value theorem D: Use the Second Fundamental Theorem of Calculus E: Use U-Substitution/Change of Variables to find integrals 1. Apply the Power Rule for integration 2. Perform Trigonometric integration F: Solve Differential Equations 1. Use separation of variables 2. Calculate Growth and Decay 3. Read and graph Slope Fields ** taught after C.	Tests Quizzes Student Presentations of homework problems at the SmartBoard Exit Slips

TEACHER RESOURCES	TEACHER NOTES/REFLECTIONS
Textbook Calculus in Motion software Graphing Calculator SMARTboard	

UNIT FIVE: APPLICATIONS OF THE INTEGRAL

UNIT SUMMARY:

As an overview:

1. Students will solve real world problems using the integral.
2. Students will apply their knowledge of integrals.
3. Students will apply their knowledge of integration to calculate area and volume in real world situations.

21ST CENTURY THEMES:

- ☒ Global Awareness
- ☒ Financial, Economic, Business and Entrepreneurial Literacy
- ☐ Civic Literacy
- ☐ Health Literacy

PRIMARY INTERDISCIPLINARY CONNECTIONS:

This unit has a direct correlation with physics, economics and engineering.

STANDARDS

N-Q 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-SSE 1. Interpret expressions that represent a quantity in terms of its context.⋆

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

2. Use the structure of an expression to identify ways to rewrite it.
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.⋆

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions.

A –APR 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A –CED 1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A -RE I 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
b. Solve quadratic equations by inspection (e.g., for $x_2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F-IF 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an Element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
8. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
b. Use the properties of exponents to interpret expressions for exponential functions.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-TF 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
9. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–x, □+x, and 2□–x in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

G-CO 1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-GMD 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

G-MG 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost;

9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences.

9.1.12.B.1 Present resources and data in a format that effectively communicates the meaning of the data and its implications for solving problems, using multiple perspectives.

9.1.12.C.5 Assume a leadership position by guiding the thinking of peers in a direction that leads to successful completion of a challenging task or project.

9.1.12.F.2 Demonstrate a positive work ethic in various settings, including the classroom and during structured learning experiences.

RST.11.3-7

3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.

5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.

6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.

7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.

UNIT ESSENTIAL QUESTIONS:	UNIT ENDURING UNDERSTANDINGS:
▪ How do we find the area between two curves? ▪ When is it applicable to integrate with respect to x or y? ▪ How do we find the volume using the disk and washer methods? ▪ How do we find the length of an arc? ▪ How do we find surface area?	▪ The integral can be used to find the area between two curves and to find the volume of solids of revolution, surface area and arc length.

TIME ALLOTMENT	CONTENT	SKILLS	ASSESSMENT
45 Days	▪ Area under a curve ▪ Disk method ▪ Washer method ▪ Shell method ▪ Arc Length ▪ Surface Area	A: Calculate the area between two curves 1. Integrate with respect to x 2. Integrate with respect to y B: Calculating Volume Using the: 1. Disk Method 2. Washer Method 3. Cross-Sections 4. Shell Method C: Calculating Arc Length D: Calculating Surface Area	Tests Quizzes Student Presentations *Pear* Lab *Bottle Design* Project

TEACHER RESOURCES	TEACHER NOTES/REFLECTIONS
Textbook *Calculus in Motion* software Graphing Calculator SMARTboard	

Geometer's Sketchpad software Centimeter graph paper Graduated cylinder Toothpicks Knives Plastic Wrap Empty bottle	